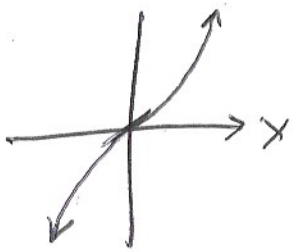
[5] $y = \sinh x$

[f]



DOMAIN $= (-\infty, \infty)$

RANGE $= (-\infty, \infty)$

NO VERTICAL NOR HORIZONTAL ASYMPTOTES

[g] 1-1 FUNCTION

FROM MATH 41

↳ FOR EACH INPUT VALUE, THERE IS AT MOST 1 OUTPUT VALUE

(VERTICAL LINE TEST)

↓

NO VERTICAL LINE CROSSES THE GRAPH MORE THAN ONCE –

EVERY VERTICAL LINE TOUCHES THE GRAPH AT MOST ONCE

↓

FOR EACH OUTPUT VALUE, THERE IS ONLY 1 CORRESPONDING INPUT VALUE

(HORIZONTAL LINE TEST)

↓

EVERY HORIZONTAL LINE TOUCHES THE GRAPH AT MOST ONCE

FROM MATH 41

$y = \sinh x$ <u>IS</u> A 1-1 FUNCTION

SO IT HAS AN INVERSE FUNCTION

$y = \sinh^{-1} x \neq \dfrac{1}{\sinh x}$

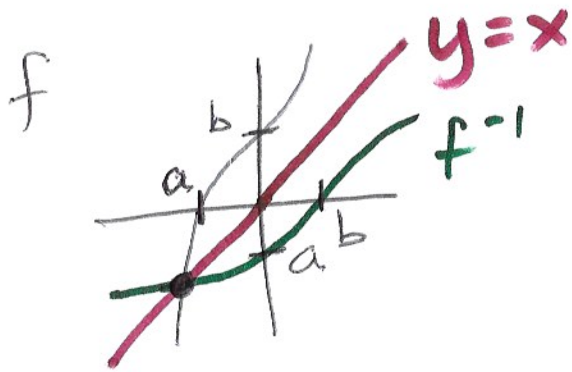
$\boxed{y = \sinh^{-1} x}$ IF AND ONLY IF $x = \sinh y$

THE INVERSE OF 1-1 FUNCTION f IS DENOTED AS f^{-1} $\left(\neq \frac{1}{f}\right)$

-1 IS NOT AN EXPONENT IF f IS A FUNCTION

FROM MATH 41:

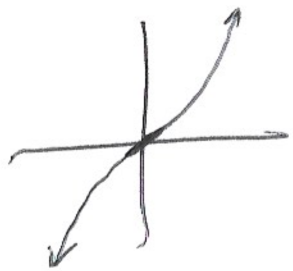
THE GRAPH OF f^{-1} IS THE GRAPH OF f REFLECTED OVER THE LINE $y=x$



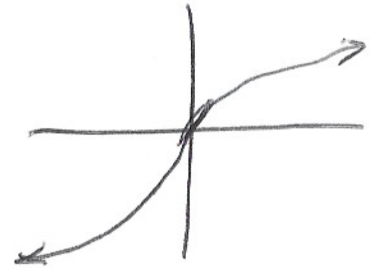
X-INT OF f = $(a, 0)$ BECOMES $(0, a)$ ON f^{-1} = y-INT OF f^{-1}

y-INT OF f = $(0, b)$ BECOMES $(b, 0)$ ON f^{-1} = X-INT OF f^{-1}

$y = \sinh x$ (USES e^x)



$y = \sinh^{-1} x$ (USES $\ln x$)



[7][b][c]

DOMAIN OF $\sinh x = (-\infty, \infty)$ = RANGE OF $\sinh^{-1} x$

↳ x ↳ y

RANGE OF $\sinh x = (-\infty, \infty)$ = DOMAIN OF $\sinh^{-1} x$

↳ y ↳ x

NO VERTICAL
NOR HORIZONTAL ASYMPTOTES

NO HORIZONTAL
NOR VERTICAL ASYMPTOTES

[6] SOLVE $\sinh x = 1$ IE, FIND $\sinh^{-1} 1 = x$

$$\frac{e^x - e^{-x}}{2} = 1$$

$$e^x - e^{-x} = 2$$

$$e^x - \frac{1}{e^x} = 2$$

LET $z = e^x$

$$z\left(z - \frac{1}{z}\right) = (2)z$$

$$z^2 - 1 = 2z$$

$$z^2 - 2z - 1 = 0$$

$a = 1$
$b = -2$
$c = -1$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot -1}}{2}$$

$$= \frac{2 \pm \sqrt{4 + 4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$z = 1 \pm \sqrt{2}$$

$$e^x = 1 \pm \sqrt{2}$$

BUT $e^x > 0$ FOR ALL $x \in \mathbb{R}$

$$1 - \sqrt{2} \approx 1 - 1.4 < 0$$

$$e^x = 1 + \sqrt{2}$$

$$x = \ln(1 + \sqrt{2})$$